# Grade 6 Math Circles
February 14/15/16, 2023
Graph Theory - Problem Set

1. Your friend says that they can draw a graph with the same number of vertices as edges. Is what your friend saying possible? Why or why not?

2. Another friend says that they can draw a bipartite graph with no edges. Is what your friend saying possible? Why or why not?

3. Suppose you have the following graph.

   (a) How many possible paths are there from vertex A to vertex B?

   (b) How many cycles are in this graph?

   (c) How many possible walks are there from vertex A to vertex B?
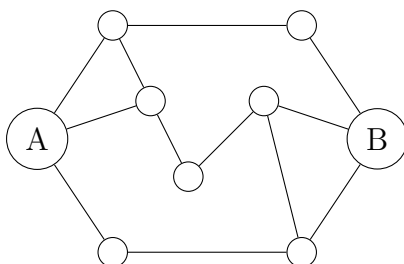
4. Show that the following graph is bipartite by separating the vertices into two partitions where a vertex in one partition does not have an edge to another vertex in the same partition. In the lesson, we learned that every bipartite graph can only contain even length cycles. Use this to check your answer.
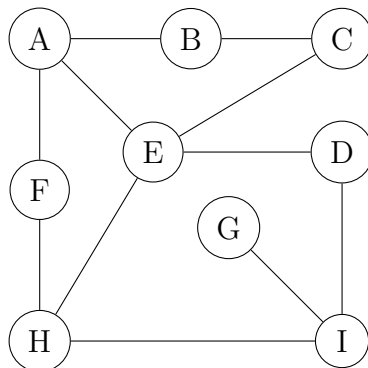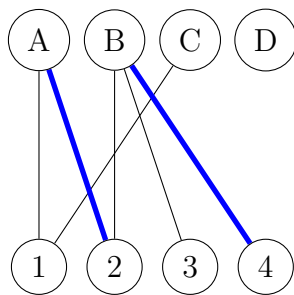

5. Given the following graphs and their follow matchings, use Berge's Theorem to show whether the matchings are maximum or not.
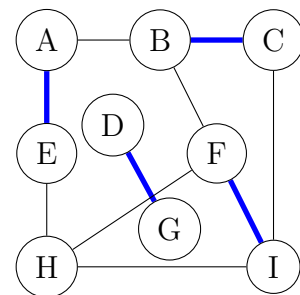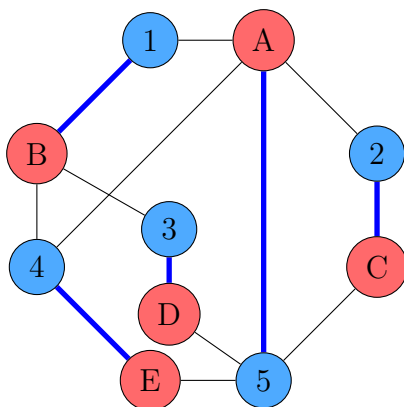
(a)
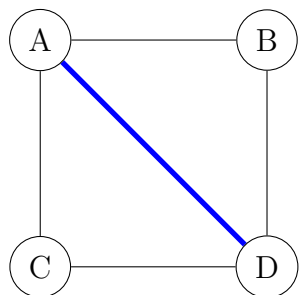
(b)

(c)

6. Given the following bipartite graph and maximum matching, use the Minimum Cover Rule to find a minimum cover.

7. Below is a bipartite graph with partitions $X = \{A, B, C, D, E, F, G, H\}$ and $Y = \{1, 2, 3, 4, 5, 6, 7, 8\}$ and a matching in blue.

   (a) Use the Hopcroft-Karp Algorithm to find a maximum matching

   (b) Apply the Minimum Cover Rule to find a minimum cover.